

## Resonance Enhanced Tunneling

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### ABSTRACT

Time evolution of tunneling in thermal medium is examined using the real-time semiclassical formalism previously developed. Effect of anharmonic terms in the potential well is shown to give a new mechanism of resonance enhanced tunneling. If the friction from environment is small enough, this mechanism may give a very large enhancement for the tunneling rate. The case of the asymmetric wine bottle potential is worked out in detail.

In our previous paper [1] we formulated within the semiclassical framework how to deal with the real-time dynamics of tunneling that takes place in thermal medium. In the present work we further analyze the problem and discover a new mechanism of enhanced tunneling caused by anharmonic terms in the potential well. The result may have great relevance to the old (once failed) scenario of inflation based on the first order phase transition [2], and to the scenario of electroweak baryogenesis [3].

We start from recapitulating our previous result [1]. The real-time dynamics of tunneling in thermal medium is studied in the standard model [4],[5], [6] of environment. Its interaction with a one dimensional system which we describe by a potential  $V(q)$  is of a bilinear form,  $q \int d\omega c(\omega)Q(\omega)$ . Here  $Q(\omega)$  is the environment oscillator coordinate of frequency  $\omega$  and  $c(\omega)$  gives a coupling strength of the system-environment interaction. With the total system thus specified, dynamics is given by the quantum equation of motion,

$$\frac{d^2q}{dt^2} + \frac{dV}{dq} = - \int_{\omega_c}^{\infty} d\omega c(\omega)Q(\omega), \quad \frac{d^2Q(\omega)}{dt^2} + \omega^2 Q(\omega) = -c(\omega)q. \quad (1)$$

Quantum Langevin equation is derived [7] by eliminating the environment variable  $Q(\omega, t)$ ;

$$\frac{d^2q}{dt^2} + \frac{dV}{dq} + 2 \int_0^t ds \alpha_I(t-s)q(s) = F_Q(t), \quad (2)$$

where  $F_Q(t)$  is linear in initial environment values,  $Q_i(\omega)$  and  $P_i(\omega)$ ,

$$F_Q(t) = - \int_{\omega_c}^{\infty} d\omega c(\omega) \left( Q_i(\omega) \cos(\omega t) + \frac{P_i(\omega)}{\omega} \sin(\omega t) \right), \quad (3)$$

and obeys the correlation formula,

$$\langle \{F_Q(\tau), F_Q(s)\}_+ \rangle_{\text{env}} = \int_{\omega_c}^{\infty} d\omega r(\omega) \cos \omega(\tau-s) \coth\left(\frac{\beta\omega}{2}\right) \equiv \alpha_R(\tau-s), \quad (4)$$

with  $r(\omega) = c^2(\omega)/(2\omega)$  and  $\beta = 1/T$  the inverse temperature. The kernel function  $\alpha_I$  in eq.(2) is given by  $\alpha_I(t) = - \int_{\omega_c}^{\infty} d\omega r(\omega) \sin(\omega t)$ . The combination,  $\alpha_R(t) + i\alpha_I(t)$ , is a sum of the real-time thermal Green's function, added with the weight  $c^2(\omega)$ .

An often used simplification is the local, Ohmic approximation taking  $r(\omega) = \eta\omega/\pi$  with  $\omega_c = 0$ , which amounts to  $\alpha_I(\tau) = \delta\omega^2\delta(\tau) + \eta\delta'(\tau)$ . This gives the local version of Langevin equation,

$$\frac{d^2q}{dt^2} + \frac{dV}{dq} + \delta\omega^2 q + \eta \frac{dq}{dt} = 0. \quad (5)$$

The parameter  $\delta\omega^2$  is interpreted as a potential renormalization or a mass renormalization in the field theory analogy, since by changing the bare frequency parameter to the renormalized  $\omega_R^2$  the term  $\delta\omega^2 q$  is cancelled by the counter term in the potential. On the other hand,  $\eta$  is the Ohmic friction coefficient. This local approximation breaks down both at early and at late times [8], but it is useful in many other cases.

The crucial observation [1] is that in the semiclassical approximation the Wigner function has the solution in terms of classical solutions;

$$f_W(q, p, Q, P; t) = \int dq_i dp_i \int dQ_i dP_i f_W^{(i)}(q_i, p_i, Q_i, P_i) \cdot \delta(q - q_{\text{cl}}) \delta(p - p_{\text{cl}}) \delta(Q - Q_{\text{cl}}) \delta(P - P_{\text{cl}}). \quad (6)$$

Here  $q_{\text{cl}}(q_i, p_i, Q_i, P_i; t)$  etc. are the solution of (1), taken as the classical equation with the specified initial condition. We take in the present work an uncorrelated initial state of the form  $\rho^{(i)} = \rho_q^{(i)} \otimes \rho_Q^{(i)}$ . The reduced Wigner function is defined after the environment variable  $(Q(\omega), P(\omega))$  integration;

$$f_W^{(R)}(q, p; t) = \int dq_i dp_i f_W^{(i)}(q_i, p_i) K(q, p, q_i, p_i; t), \quad (7)$$

$$K(q, p, q_i, p_i; t) = \int dQ_i dP_i f_W^{(i)}(Q_i, P_i) \delta(q - q_{\text{cl}}) \delta(p - p_{\text{cl}}). \quad (8)$$

The next important point is expansion of the classical solution  $q_{\text{cl}}$  in the environment initial variables  $(Q_i, P_i)$ ;

$$q_{\text{cl}}(t) \approx q_{\text{cl}}^{(0)}(t) + \int d\omega \left\{ Q_i(\omega) q_{\text{cl}}^{(Q)}(\omega, t) + P_i(\omega) q_{\text{cl}}^{(P)}(\omega, t) \right\}. \quad (9)$$

The truncation up to the first order term in  $(Q_i(\omega), P_i(\omega))$  should be adequate at low temperatures. With the help of the familiar Fourier formula for the delta function in eq.(8), one derives, using the Gaussian thermal function for the initial  $f_W^{(i)}(Q_i, P_i)$ , an integral transform of the Wigner function,  $f_W^{(i)}(q_i, p_i) \rightarrow f_W^{(R)}$ . The resulting kernel function is given by [1]

$$K(q, p, q_i, p_i; t) = \frac{\sqrt{\det \mathcal{J}}}{2\pi} \exp \left[ -\frac{1}{2} (q - q_{\text{cl}}^{(0)}, p - p_{\text{cl}}^{(0)}) \mathcal{J} \begin{pmatrix} q - q_{\text{cl}}^{(0)} \\ p - p_{\text{cl}}^{(0)} \end{pmatrix} \right]. \quad (10)$$

The matrix elements of  $(\mathcal{J}^{-1})_{ij} = I_{ij}$  are given as follows. First,

$$I_{11} = \frac{1}{2} \int_{\omega_c}^{\infty} d\omega \coth \frac{\beta\omega}{2} \frac{1}{\omega} |z(\omega, t)|^2, \quad (11)$$

and  $I_{22}$  is given by a similar integral, replacing  $z(\omega, t)$  in  $I_{11}$  by  $\dot{z}(\omega, t)$ , while  $I_{12} = \frac{\dot{I}_{11}}{2I_{11}}$ . Here  $z(\omega, t) = q_{\text{cl}}^{(Q)}(\omega, t) + i\omega q_{\text{cl}}^{(P)}(\omega, t)$ , and  $\dot{z}(\omega, t) = p_{\text{cl}}^{(Q)}(\omega, t) + i\omega p_{\text{cl}}^{(P)}(\omega, t)$ .

Quantities that appear in the integral transform are determined by solving differential equations; the homogeneous Langevin equation for  $q_{\text{cl}}^{(0)}$  and an inhomogeneous linear equation for  $z(\omega, t)$  and  $\dot{z}(\omega, t)$ ,

$$\frac{d^2 q_{\text{cl}}^{(0)}}{dt^2} + \left( \frac{dV}{dq} \right)_{q_{\text{cl}}^{(0)}} + 2 \int_0^t ds \alpha_I(t-s) q_{\text{cl}}^{(0)}(s) = 0, \quad (12)$$

$$\frac{d^2 z(\omega, t)}{dt^2} + \left( \frac{d^2 V}{dq^2} \right)_{q_{\text{cl}}^{(0)}} z(\omega, t) + 2 \int_0^t ds \alpha_I(t-s) z(\omega, s) = -c(\omega) e^{i\omega t}. \quad (13)$$

A similar equation as for  $z(\omega, t)$  holds for  $\dot{z}(\omega, t)$ . The initial condition is  $q_{\text{cl}}^{(0)}(t=0) = q_i$ ,  $p_{\text{cl}}^{(0)}(t=0) = p_i$ ,  $z(\omega, t=0) = 0$ ,  $\dot{z}(\omega, t=0) = 0$ .

The physical picture underlying the formula for the integral transform, eq.(7) along with (10), should be evident; the probability at a phase space point  $(q, p)$  is dominated by the semiclassical trajectory  $q_{\text{cl}}^{(0)}$  (environment effect of dissipation being included for its determination by eq.(12)) reaching  $(q, p)$  from an initial point  $(q_i, p_i)$  whose contribution is weighed by the quantum mechanical probability  $f_W^{(i)}$  initially given. The contributing trajectory is broadened by the environment interaction with the width factor  $\sqrt{I_{ij}}$ . The quantity  $I_{11}$  given by (11) and (13), for instance, is equal to  $\overline{(q - q_{\text{cl}}^{(0)})^2}$ ; an environment driven fluctuation under the stochastic force  $F_Q(t)$ .

The tunneling potential is divided into two regions separated at the position  $q = q_B$  of the barrier top. In the present work we shall assume that the potential is very steep at both ends;  $V(\pm\infty) = \infty$ . The tunneling rate, from the inner region at  $q < q_B$  called here the potential well into the outer region at  $q > q_B$ , is an important measure of tunneling phenomena and is given by the flux at  $q = q_B$ ;  $\dot{P}(t) = -I(q_B, t)$ , which is equal to

$$- \int dq_i dp_i f_W^{(i)}(q_i, p_i) \left( p_{\text{cl}}^{(0)} + \frac{\dot{I}_{11}}{2I_{11}} (q_B - q_{\text{cl}}^{(0)}) \right) \frac{1}{\sqrt{2\pi I_{11}}} \exp \left[ -\frac{(q_B - q_{\text{cl}}^{(0)})^2}{2I_{11}} \right]. \quad (14)$$

The width factor  $\sqrt{I_{11}}$  thus determines the contributing region of  $q_{\text{cl}}^{(0)}$  by how much away it is from  $q_B$ . On the other hand, the tunneling probability into the overbarrier region at  $q > x$  is given by

$$P(x, t) = \int dq_i dp_i f_W^{(i)}(q_i, p_i) \int_x^\infty du \frac{1}{\sqrt{2\pi I_{11}}} \exp \left[ -\frac{(u - q_{\text{cl}}^{(0)})^2}{2I_{11}} \right]. \quad (15)$$

In both of the quantities,  $\dot{P}(t)$  and  $P(q_B, t)$  the tunneling probability into  $q > q_B$ , it is essential to estimate how  $q_{\text{cl}}^{(0)}$  and  $I_{11}$  varies with time.

We shall assume an initial state localized in the potential well so that the dominant contribution in the  $(q_i, p_i)$  phase space integration is restricted to  $q_i < q_B$ . Let us first consider the harmonic approximation for the potential well region given by  $V(q) + \frac{1}{2} \delta \omega^2 q^2 \approx \frac{1}{2} \omega_0^2 q^2$  near the bottom of the well at  $q = 0$ , and use the Ohmic friction. The approximate solution is given by

$$q_{\text{cl}}^{(0)} = \left( \cos \tilde{\omega}_0 t + \frac{\eta}{2\tilde{\omega}_0} \sin \tilde{\omega}_0 t \right) e^{-\eta t/2} q_i + \frac{\sin \tilde{\omega}_0 t}{\tilde{\omega}_0} e^{-\eta t/2} p_i, \quad (16)$$

$$z(\omega, t) = \frac{c(\omega)}{\omega^2 - \omega_0^2 - i\omega\eta} \cdot \left( e^{i\omega t} - \frac{\omega + \tilde{\omega}_0 - i\eta/2}{2\tilde{\omega}_0} e^{i\tilde{\omega}_0 t - \eta t/2} + \frac{\omega - \tilde{\omega}_0 - i\eta/2}{2\tilde{\omega}_0} e^{-i\tilde{\omega}_0 t - \eta t/2} \right), \quad (17)$$

using  $\tilde{\omega}_0 = \sqrt{\omega_0^2 - \frac{\eta^2}{4}}$ . We assume a small friction,  $\eta \ll \omega_0$ .

Near  $\omega = \tilde{\omega}_0$  this function is approximately

$$z(\omega, t) \approx \frac{i c(\omega)}{\omega + \tilde{\omega}_0 - i\eta/2} \left( t e^{i\tilde{\omega}_0 t} - \frac{1}{\tilde{\omega}_0} \sin \tilde{\omega}_0 t \right) e^{-\eta t/2}. \quad (18)$$

This formula is valid at  $t < O[1/\eta]$ . The appearance of the linear  $t$  term is a resonance effect. The resonance roughly contributes to  $I_{11}(t)$  by the amount,  $\eta t^2 e^{-\eta t} \times$  a smooth  $\omega$  integral which is cutoff by a physical frequency scale. Thus, the width factor  $I_{11}$  initially increases with time until the time scale of order  $1/\eta$ .

At large times the classical trajectory  $q_{\text{cl}}^{(0)}$  asymptotically approaches towards  $q = 0$ , the local minimum of the potential. The width factor behaves as

$$I_{11}(t) = I_{11}(\infty) + O[e^{-\eta t/2}], \quad (19)$$

$$I_{11}(\infty) \approx \frac{1}{2\omega_0} + \frac{1}{\omega_0} \frac{1}{e^{\omega_0/T} - 1} + \frac{\pi}{3} \frac{\eta}{\omega_0^4} T^2. \quad (20)$$

The asymptotic value of  $I_{11}(\infty)$  has the familiar zero point fluctuation of harmonic oscillator and in the last term the dominant finite temperature correction, valid for this Ohmic model at  $T \ll \omega_0$ . In any event the probability rate  $\dot{P}(t)$  finally decreases to zero, along with

$$p_{\text{cl}}^{(0)} + \frac{\dot{I}_{11}}{2I_{11}} (q_B - q_{\text{cl}}^{(0)}) \rightarrow 0. \quad (21)$$

Moreover, the final tunneling probability  $P(q_B, \infty)$  has a finite value, and typically is very small for a large potential barrier. For instance, for the asymmetric wine bottle potential later discussed,

$$P(q_B, \infty) \approx \frac{1}{4} \sqrt{\frac{\omega_0}{2\pi V_h}} e^{-8V_h/\omega_0}, \quad (22)$$

with  $V_h$  the barrier height much smaller than  $\omega_0$ . This poses a curious question; it appears that decay of a prepared metastable state localized in the potential well is never completed.

This simple picture is however valid only when one ignores anharmonic terms in the tunneling potential, but they must be there in order to give any realistic tunneling potential. The most important point for the present work is effect of anharmonic terms in the equation for the fluctuation  $z(\omega, t)$ . Presence of anharmonic terms gives a non-trivial periodicity in the coefficient function  $\left(\frac{d^2V}{dq^2}\right)_{q_{\text{cl}}^{(0)}}$  for (13), assuming a small friction  $\eta \ll \omega_0$ . For a very small friction the anharmonic term becomes important both for completion of the decay and the new mechanism of resonance enhanced tunneling. In the rest of this paper we shall discuss the mechanism of resonance enhancement.

One might naively expect that the homogeneous part of the  $z(\omega, t)$  solution exhibits the well known parametric resonance [9], if the relevant parameter in the periodic coefficient function  $\left(\frac{d^2V}{dq^2}\right)_{q_{\text{cl}}^{(0)}}$  falls in the instability band. But this is not what happens, as will be shown. Thus, unbounded exponential growth of  $\sqrt{T_{11}}$  does not take place. On the other hand, the power-law growth is observed, as clearly seen in numerical computation. Moreover, we find that our  $z(\omega, t)$  solution belongs to the boundary between stability and instability bands. At the resonance frequency the enhancement factor due to the boundary effect is much larger than what one might expect from the harmonic case (18). We shall interpret this phenomenon as influenced in a subtle way by the parametric resonance, although it is not the parametric resonance itself.

Observe first that if  $q_{\text{cl}}^{(0)}(t; q_i, p_i)$  is a solution for the classical, homogeneous Langevin equation, then  $q_{\text{cl}}^{(0)}(t + \epsilon; q_i, p_i)$  for an arbitrary constant time shift  $\epsilon$  is also a solution. Hence the time derivative  $\dot{q}_{\text{cl}}^{(0)}(t; q_i, p_i) = p_{\text{cl}}^{(0)}(t; q_i, p_i)$  is a homogeneous solution for the linearized  $z(\omega, t)$  equation. Using this homogeneous solution, one can readily find the inhomogeneous solution for  $z(\omega, t)$  with the initial condition,  $z(\omega, 0) = \dot{z}(\omega, 0) = 0$ . Corresponding to the classical motion  $q_{\text{cl}}^{(0)}$  of given initial condition  $(q_i, p_i)$ , this solution is

$$z(\omega, t) = -c(\omega) p_{\text{cl}}^{(0)}(t) \int_0^t dt' \left(p_{\text{cl}}^{(0)}(t')\right)^{-2} \int_0^{t'} dt'' e^{-\eta(t'-t'')} p_{\text{cl}}^{(0)}(t'') e^{i\omega t''}. \quad (23)$$

It is best to discuss the resonance enhanced tunneling mechanism in concrete examples. We take the asymmetric wine bottle potential as illustrated in Fig.1,

which is described in the well and its vicinity region by

$$V(q) \approx \frac{\lambda}{4}(q^2 - 2q_B q)^2. \quad (24)$$

The curvature parameters at two extrema of  $q = 0$  and  $q = q_B$  are  $\omega_0^2 = 2\lambda q_B^2$ ,  $\omega_B^2 = \lambda q_B^2$ , and the barrier height seen from the bottom of the well is  $V_h = \frac{\lambda}{4}q_B^4 = \omega_0^2 q_B^2/8$ . The condition for a large barrier,  $V_h \gg \omega_0$ , implies  $\sqrt{\lambda} q_B^3 \gg 4\sqrt{2}$ . The classical  $q_{\text{cl}}^{(0)}$  equation in the Ohmic approximation is written using rescaled variables,  $y = q_{\text{cl}}^{(0)}/q_B$  and  $\tau = \omega_0 t/2$ ,

$$y'' + y(y-1)(y-2) + \frac{2\eta}{\omega_0} y' = 0, \quad (25)$$

where  $y' = dy/d\tau$ .

To give an idea of the magnitude of initial amplitudes,  $y(0)$  and  $y'(0)$ , let us assume an initial thermal state trapped in the (hypothetical) harmonic potential well, taking the same temperature as the environment. The magnitude of initial values is then on the average  $\sqrt{\overline{q_i^2}} = \sqrt{\coth \frac{\omega\omega_0}{2}/(2\omega_0)} \approx \sqrt{1/(2\omega_0)}$ ,  $\sqrt{\overline{p_i^2}} \approx \sqrt{\omega_0/2}$ . This gives

$$\frac{\sqrt{\overline{q_i^2}}}{q_B} = \frac{\sqrt{\overline{p_i^2}}}{\omega_0 q_B} = \frac{1}{4} \sqrt{\frac{\omega_0}{V_h}}. \quad (26)$$

The classical solution  $p_{\text{cl}}^{(0)}$  that appears in the  $z(\omega, t)$  solution (23) is given by the Jacobi's elliptic function for the symmetric double well, the situation relevant to the case of a small friction. This function has the fundamental period given by

$$T = \frac{4}{\omega_0 \sqrt{1 + \sqrt{\epsilon}}} \int_0^1 \frac{du}{\sqrt{(1-u^2)(1-k^2 u^2)}}, \quad k = \sqrt{\frac{2\sqrt{\epsilon}}{1 + \sqrt{\epsilon}}}, \quad (27)$$

where  $\epsilon = E_i/V_h$ ,  $E_i = p_i^2/2 + \omega_0^2 q_i^2/2$ . The corresponding fundamental frequency  $\omega_*$  is  $2\pi/T$ . The parameter  $k$ , hence  $\epsilon$  is a measure of the anharmonicity. For instance,  $E(q_i, p_i) = V_h/4$  corresponds to  $k \approx 0.80$ , a value giving  $\omega_*$  different from the harmonic frequency  $\omega_0$  at the potential bottom;  $\omega_* \approx 0.95 \times \omega_0$ . These formulas of the period are used to later determine the band structure associated with the  $z(\omega, t)$  equation.

We numerically integrated the coupled  $q_{\text{cl}}^{(0)}$  and  $z(\omega, t)$  equations. In the  $\omega$  integral (11) for the width factor  $I_{11}$  the largest contribution is found to come from the resonance at  $\omega = \omega_*$ , then contribution from higher harmonics at  $n\omega_*$  follows. Thus, phenomenon of a non-linear resonance occurs. In Fig.2 we show the width factor as a function of time for a few choices of the initial data  $(q_i, p_i)$ . For simplicity an

equi-partition of the kinetic and the potential energy  $p_i^2/2 = \omega_0^2 q_i^2/2$  is assumed to reduce relevant parameter dependence. The result for the width factor is plotted for a few choices of the initial energy. At an intermediate time of  $O[1/\eta]$  the width factor becomes maximal at its value much larger than in the harmonic case. The decrease at late times is due to the friction; we explicitly checked that for the zero friction, the fluctuation  $|z(\omega, t)/c(\omega)|^2$  increases in time without a bound, with an averaged time power close to 4 at the resonance. Note that even if the effect of the friction is turned off, there exists an important environment effect here; the environment interaction drives the non-linear resonance oscillation.

The unbounded power-law increase suggests that the relevant parameter in the homogeneous  $z(\omega, t)$  equation falls in the boundary between the stability and the instability bands, since otherwise it either is bounded or grows exponentially. We indeed checked this numerically by arbitrarily changing the parameters in the  $z(\omega, t)$  equation. We used the homogeneous equation introducing two arbitrary parameters  $(h, \theta)$ ;

$$z'' + \left( h - 2\theta \left( y - \frac{1}{2} y^2 \right) \right) z = 0. \quad (28)$$

Unlike a single, linear equation for the Mathieu type our problem is a coupled, non-linear system, with  $y$  to be determined by eq.(25) for  $\eta = 0$ . We must further assume some initial values for  $y(0)$ ,  $y'(0)$ . The standard algorithm that determines the stability and the instability band is used [10]. The parameter set  $(h, \theta) = (4, 3)$  corresponds to our homogeneous  $z(\omega, t)$  equation for the zero friction. This case is seen right on the boundary line of the two bands as depicted in Fig.3. We checked that our system is always on the boundary line for all choices of  $y(0)$ ,  $y'(0)$  we computed for.

A rough analytic understanding of the growth of the width factor seems to be as follows. In the explicit formula for  $z(\omega, t)$ , eq.(23), one may replace the quantity  $\left( p_{\text{cl}}^{(0)}(t') \right)^{-2}$  in the integrand by some constant average value. For a small friction it is then easy to obtain a linearly growing  $z(\omega, t)$  off the resonance and a quadratically growing  $z(\omega, t)$  right on the resonance as a function of time in the range  $t < O[1/\eta]$ . This time dependence is what is observed in numerical computation.

Computation of the tunneling rate  $\dot{P}(t)$  is a demanding task of numerical calculation, and it is left to future work. But a general trend is expected already in the following simplified computation. The most important part for  $\dot{P}(t)$  is two competing exponential factors, the one for the initial state and the other for the kernel



factor;

$$A = \exp \left[ - \tanh \frac{\beta \omega_0}{2} \frac{p_i^2 + \omega_0^2 q_i^2}{\omega_0} \right] \times \exp \left[ - \frac{(q_B - q_{cl}^{(0)})^2}{2I_{11}} \right]. \quad (29)$$

We plotted in Fig.4 the time evolution of this product. The time averaged evolution (averaged over a short time scale of several times the fundamental period) shows a rapid rise of the product factor, reflecting the initial behavior,  $I_{11} \propto t^4$ . The product then reaches some maximum at a time of order  $1/\eta$ . The asymptotic time limit coincides with that given by the harmonic approximation. As a numerical guide note that  $e^{-q_B^2/(2I_{11})} = e^{-(2V_h/\omega_0)(\omega_0 I_{11}/2)^{-1}} = e^{-20(\omega_0 I_{11}/2)^{-1}}$  for  $V_h = 10\omega_0$ .

In Fig.5 we show for a few choices of the friction the maximum value of  $A$  as a function of the initial energy  $E_i$ , assuming the equi-partition  $p_i = \omega_0 q_i$ . Remarkably, the largest contribution comes, not from the dominant initial component near the zero point energy, rather from the initially suppressed excited component. The first exponent in (29) is in proportion to  $E_i$ , while the second one goes roughly like  $E_i^{-2}$ , hence a maximum may appear somewhere away from the lowest energy of  $E_i = \omega_0/2$ . In this example of  $\eta/\omega_0 = 0.0025$  the maximum product factor is of order  $10^{-3}$  at  $E_i \approx 4 \times \omega_0/2$ , 6 orders of magnitudes larger than what one expects from the lowest energy state and also the asymptotic value of order  $10^{-36}$ . A large value of the product factor of order unity suggests an interesting possibility of a rapid and violent termination of the tunneling.

We end with some speculative comments on impact of the resonance enhanced tunneling when applied to cosmology. For this part of discussion we assume that the non-linear resonance we found in the present work terminates the tunneling associated with the asymmetric wine bottle type of potential. The tunneling phenomenon is clearly related to termination of the first order phase transition, if there exists a prolonged period of cooling providing a chance that the universe, or a part of it, is trapped in a supercooled metastable state localized, say near  $q = 0$  in our quantum mechanical terminology. The coordinate  $q$  here should be understood as the order parameter of the phase transition, the homogeneous Higgs field in the case of GUT [2] and electroweak [3] phase transition. The environment in this case is made of various forms of matter fields, the quark and the lepton fields, and also the gauge and the Higgs particle. In the usual weak coupling theory the friction containing typically small Yukawa couplings is small compared to the typical energy scale of the system, the Higgs mass. Thus the small dissipation seems a good approximation. The low temperature approximation also works if  $T \ll$  the Higgs mass. After

many periods of the periodic Higgs motion (the period  $\approx 2\pi/(\text{Higgs mass})$ ) the resonance amplification is expected to lead to an outburst of tunneling. This may occur stochastically in different parts of the universe, leaving behind nucleated bubbles, although it is not clear how different bubbles subsequently merge. Since the nucleation occurs presumably within a short time in all parts of the universe, it appears that the first order phase transition is terminated abruptly. This highly non-equilibrium incident should much help the electroweak baryogenesis.

The other possibility of completing the phase transition is signaled by the change of the potential (more properly of the free energy) due to the temperature change so that the once local minimum at  $q = 0$  becomes an unstable, local maximum of the potential. Under a normal circumstance the rate of the temperature change is given by the Hubble parameter  $H$ , hence if the Hubble parameter is small enough, like  $H < \eta$  (the friction), completion of the phase transition via the resonance enhanced tunneling most likely takes place. This condition seems to be satisfied in the two cosmological examples given.

In summary, we gave using the real-time formalism of tunneling dynamics a new mechanism of resonance enhanced tunneling.

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## Figure caption

### Fig.1

A schematic form of asymmetric wine bottle potential. A case of large barrier  $V_h = 10 \omega_0$  is depicted. The barrier top is at  $q_B = 4\sqrt{5}/\sqrt{\omega_0}$ .

### Fig.2

Time evolution of the width factor  $I_{11}$ . A time average over  $\omega_0 \Delta t / 2 = 20$  of the dimensionless quantity  $\omega_0 I_{11} / 2$  is plotted as a function of time  $\times \omega_0 / 2$ . Examples of initial states of energy  $E_i = (1, 2, 5) \times \omega_0 / 2$  ( $\omega_0 / 2$  being the ground state energy in the potential well) are compared to the harmonic case. Assumed parameters are  $V_h = 10 \omega_0$ ,  $\eta / \omega_0 = 0.0025$ ,  $T / \omega_0 = 1/20$ ,  $\Omega(\text{cutoff}) / \omega_0 = 25$ .

### Fig.3

Band structure of stability (unshaded) and instability (shaded) for the modified fluctuation equation. The defining equation is (28), along with (25), with the initial condition of  $y(0) = y'(0) = 0.2$ . The point marked by the cross has  $(h, \theta) = (4, 3)$  corresponding to our system.

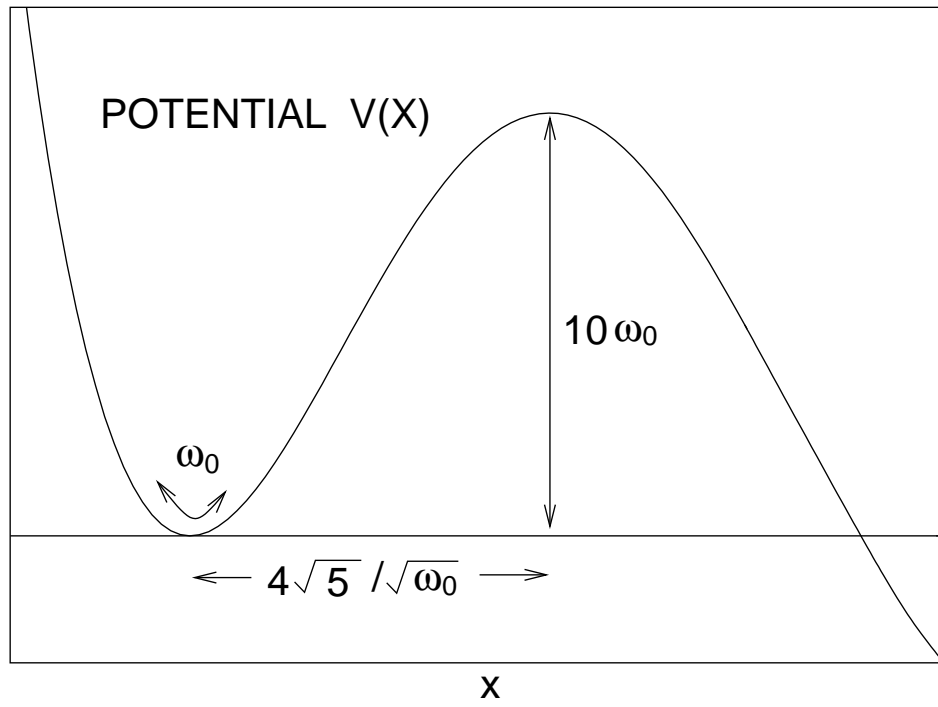
### Fig.4

Time averaged evolution of the product  $A$  of two exponential factors for the tunneling rate. The same parameter set as in Fig.2 is taken. In the inset the case of  $E_i = 5 \times \omega_0 / 2$  is depicted in the linear scale.

### Fig.5

Maximal value of the product  $A$  of two exponential factors for the tunneling rate is plotted as a function of the initial energy for a few choices of the friction  $\eta$ . The other parameters are taken the same as in Fig.2.

Fig. 1



$\langle \omega_0 I_{11} / 2 \rangle$

Fig. 2

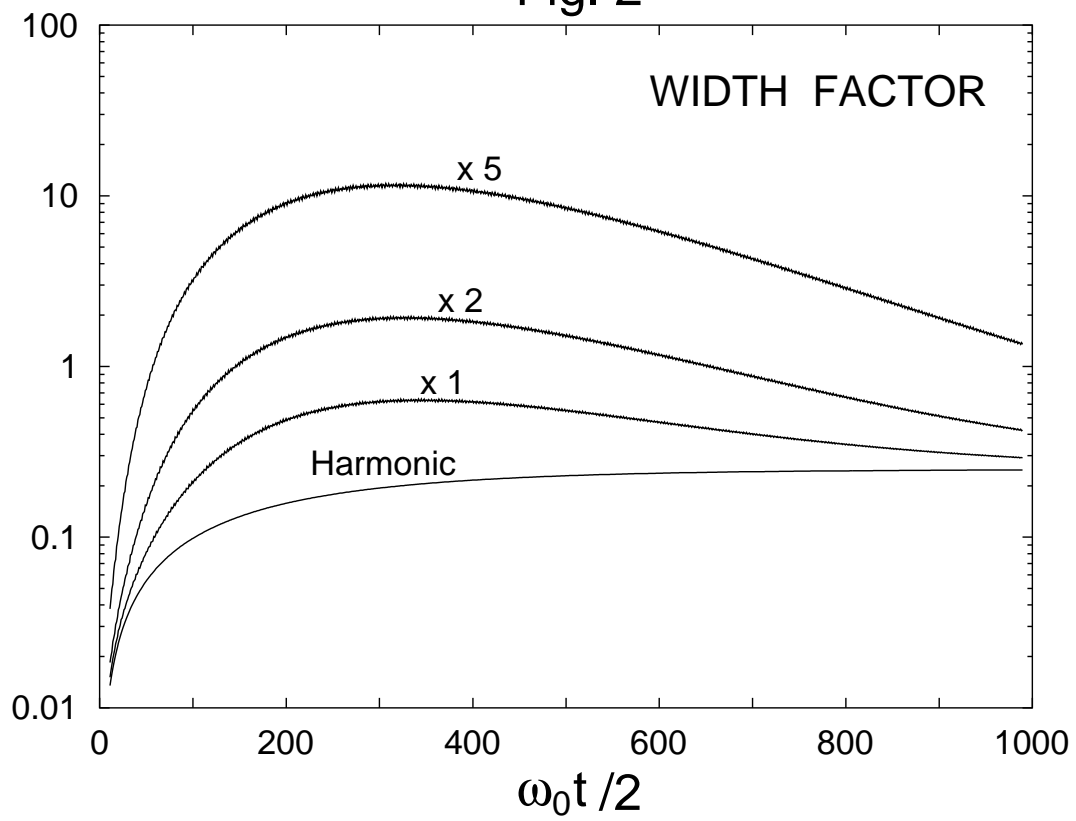


Fig. 3

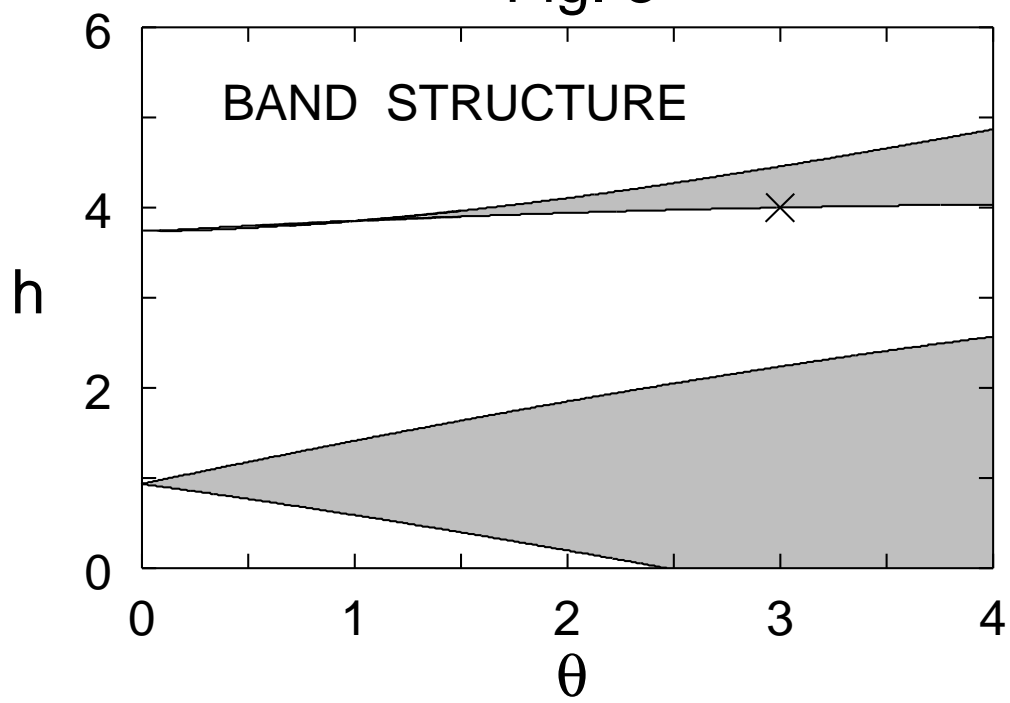


Fig. 4

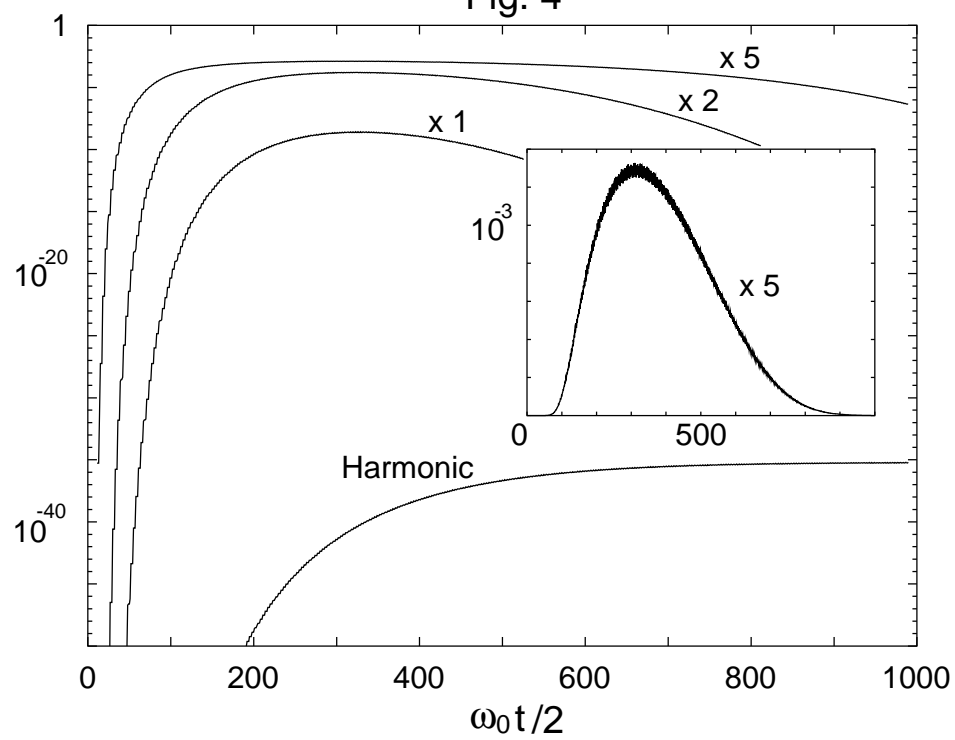


Fig. 5

